INTERNATIONAL STANDARD ISO 2975/I

INTERNATIONAL ORGANIZATION FOR STANDARDIZATION •МЕЖДУНАРОДНАЯ ОРГАНИЗАЦИЯ ПО СТАНДАРТИЗАЦИИ•ORGANISATION INTERNATIONALE DE NORMALISATION

Measurement of water flow in closed conduits – Tracer methods – Part I : General

Mesure de débit de l'eau dans les conduites fermées – Méthodes par traceurs –
Partie I : Généralités

First edition – 1974-05-01

UDC 681.121.84

Ref. No. ISO 2975/I-1974 (E)

Descriptors : liquid flow, water flow, pipe flow, flow measurement, isotopic labelling.

Price based on 11 pages

FOREWORD

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Draft International Standards adopted by the Technical Committees are circulated to the Member Bodies for approval before their acceptance as International Standards by the ISO Council.

International Standard ISO 2975/I was drawn up by Technical Committee ISO/TC 30, *Measurement of fluid flow in closed conduits*, and circulated to the Member Bodies in October 1972.

It has been approved by the Member Bodies of the following countries :

Australia
Belgium
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Germany
Hungary
Japan
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New Zealand
South Africa, Rep. of
Spain
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United Kingdom
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No Member Body expressed disapproval of the document.

Printed in Switzerland

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Measurement of water flow in closed conduits — Tracer methods — Part I : General

0 INTRODUCTION

This International Standard is the first of a series of standards covering tracer methods of water flow measurement in closed conduits. The complete series of standards will be as follows :

- Part I : *General.*
- Part II : *Constant rate injection method using non-radioactive tracers.*
- Part III : *Constant rate injection method using radioactive tracers.*
- Part IV : *Integration (sudden injection) method using non-radioactive tracers.*
- Part V : *Integration (sudden injection) method using radioactive tracers.*
- Part VI : *Transit time method using non-radioactive tracers.*
- Part VII : *Transit time method using radioactive tracers.*

1 SCOPE AND FIELD OF APPLICATION

This International Standard deals with the measurement of water flow in closed conduits by using tracer methods; the flow of other liquids and of gases will be dealt with in subsequent International Standards.

These methods apply to flow measurement in conduits into which a solution can be injected and those cases where effective mixing of this solution with the water flowing in the conduit can be achieved, this last condition being fundamental.

Three fundamental procedures are used :

> Two procedures, known as the constant rate injection method and the integration (sudden injection) method, are based on the dilution principle : a tracer solution is injected into the conduit and the dilution (ratio) of this tracer in the water flowing in the conduit is determined, this dilution being proportional to the rate of flow.
>
> The third is a method of measurement of the mean transit time (formerly called Allen velocity method) : the tracer is injected into the conduit and the time taken by the tracer to travel a specified length between two cross-sections in each of which it is detected, is measured.

In these three methods, the advantages and disadvantages of which are considered in clause 4, the distance between injection and measuring sections should be great enough to achieve adequate mixing of the tracer with the water flowing in the conduit; the problem of an adequate mixing distance is considered in clause 6.

A large number of different tracers may be used, such as radioactive or non-radioactive, mineral or organic materials. The choice of tracer depends on the circumstances of the measurement (clause 5). The error in measurements using tracers can be less than 1 % under good conditions (clause 7).

2 VOCABULARY AND SYMBOLS

The vocabulary and symbols used in this International Standard will be defined in ISO ..., *Glossary of terms and symbols relative to the measurement of fluid flow in closed conduits.*[1]

3 UNITS

The basic units in this International Standard are SI units.

4 CHOICE OF METHOD

4.1 Comparison between dilution methods and methods based on transit time measurement

4.1.1 *Advantage of the dilution methods*

It is not necessary to know the geometrical characteristics of the conduit.

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[1] In preparation.

4.1.2 *Advantages of methods based on transit time measurement*

It is necessary only to determine the concentration-time distribution at two measuring cross-sections separated by a known volume of pipe. It is not necessary to know volumes, masses, rates of flow or characteristics of the injected solutions.

4.2 Comparison between the two dilution methods (constant rate injection and integration (sudden injection) procedure)

4.2.1 *Advantages of the constant rate injection method*

- If the rate of flow of the injection is known to be of the required accuracy and constancy it is not necessary to measure any period of injection.
- It is simple to check good mixing by using only one instrument when samples can be taken at different locations in the plane of the points of measurement. However, several instruments in parallel should be used when the same verification is required in the integration method.
- It is simpler to determine random errors.
- It is not necessary to know the volume of injected solution.

4.2.2 *Advantages of the integration method*

- This method requires a smaller mass of tracer and less time than the constant rate injection method.
- Method of injection does not matter and the apparatus is simple.
- With the same quantity of tracer it is possible to make the measurement over a greater length of pipe.

5 CHOICE OF TRACER

5.1 General

A large number of different tracers may be used, such as radioactive or non-radioactive, mineral or organic, but it is necessary for any tracer to comply with the following requirements :

a) it should mix easily with water;

b) it should cause only negligible modifications of the rate of flow;

c) it should be detectable at a concentration lower than the highest permissible concentration while taking account of toxicity, corrosion, etc.

It is also preferable for the tracer to comply with the following requirements :

d) it should be cheap;

e) it should only be present in the water flowing in the conduit at a negligible or constant concentration.

In addition, for dilution methods, it is important that the tracer

f) can, at low concentrations, be analysed accurately;

g) should not react with the water flowing in the conduit or with any other substance with which it may come into contact in such a way as to affect the measurement.

Furthermore, for transit time methods, it is absolutely necessary that the tracer be such that

h) its concentration in the measuring cross-sections can be determined at any moment.

The following substances are given as examples, together with final minimum concentrations at which they can be detected at the required levels after dilution in water :

5.1.1 *Non-radioactive tracers*

- sodium dichromate ($Na_2Cr_2O_7{\cdot}2H_2O$) :
  2×10^{-1} mg/l with direct analysis
  2×10^{-3} mg/l after reconcentration
- sodium chloride (NaCl) : 1 to 10 mg/l depending on original conductivity
- rhodamine B ($C_{28}H_{31}ClN_2O_3$) : 2×10^{-4} mg/l
- rhodamine Wt
- lithium chloride (LiCl)
- fluorescein ($C_{20}H_{10}O_5Na_2$) : 5×10^{-3} mg/l

Other tracers have been used, and in particular :

- sodium nitrite ($NaNO_2$)
- manganese sulphate ($MnSO_4{\cdot}4H_2O$)
- sulfo-rhodamine G

5.1.2 *Radioactive tracers*

Dilution and transit time methods :

- bromine 82 (half-life 36 h, γ energies of 0,55 to 1,48 MeV)
- sodium 24 (half-life 15 h, γ energies of 1,37 to 2,75 MeV)

Other isotopes such as :

- gold 198 (half-life 2,7 days, γ energies of 0,41 to 1,09 MeV)

- iodine 131 (half-life 8,04 days, γ energies of 0,25 to 0,81 MeV)
- chromium 51 (half-life 27,8 days, γ energy of 0,32 MeV)

may be used if preliminary measurements confirm that no adsorption of tracer occurs on the walls of the conduit or on the walls of sampling and counting containers.

Tritium (half-life 12,26 years, β energy of 0,018 MeV) may also be used.

In addition, for transit time methods, because the effect of wall adsorption on the measurement is not as great as in the dilution methods, other isotopes may be used, in particular, isotopes obtained from radioactive cows, such as :

- 137 caesium–137 m barium (half-life 2,6 min, γ energy of 0,66 MeV)
- 113 tin–113 m indium (half-life 104 min, γ energy of 0,39 MeV)

5.2 Advantages or disadvantages of the different tracers

5.2.1 *Comparison between radioactive and non-radioactive tracers*

5.2.1.1 ADVANTAGES OF RADIOACTIVE TRACERS

- These can be detected by means of probes located outside the conduit (for tracers emitting γ radiation).
- The measurements are less affected by turbidity of the water than are those made with non-radioactive tracers.
- If the basic substance of tracers with a short half-life is inoffensive, any contamination danger disappears very quickly and there is no permanent pollution.
- The cost of the tracer is not proportional to the rate of flow to be measured, and this makes its use attractive in the case of large rates of flow.

5.2.1.2 PARTICULAR ADVANTAGES OF RADIO-ELEMENT GENERATORS

- A practically inexhaustible quantity of radioactive tracer of short half-life is availabe at the measuring place from a small quantity of "mother" substance of long half-life at a very low cost.
- It is possible to make repeated measurements in recirculating systems when the "daughter" substance has a sufficiently short half-life.

5.2.1.3 ADVANTAGES OF NON-RADIOACTIVE TRACERS

- It is not necessary for the operators to be specially trained and classified.
- Substances generally remain stable with time; delays and distances between the supply and the use of the substance do not matter.
- Transport and injection of the substance do not require particular safety procedures; containers can be light.
- Administrative authorizations are not necessary for each measurement.

5.2.2 *Comparison of various non-radioactive tracers in general use*

5.2.2.1 ADVANTAGES OF DICHROMATE

- Small concentrations can be analyzed by colorimetric procedures which do not require specialised operators; it can easily be reconcentrated.
- It is not present in natural waters.
- It is very stable when in crystalline form and relatively stable when in solution in pure waters, even in sunshine.
- It is very soluble in water (more than 600 g/l).
- It is relatively cheap.

5.2.2.2 DISADVANTAGES OF DICHROMATE

- Instability in certain reducing conditions.
- Need of a reagent.
- Toxic in concentrated solutions.

5.2.2.3 ADVANTAGES OF RHODAMINES

- They can be analyzed in smaller concentrations than can dichromate.
- Their concentration can be recorded during measurements as they do not need any reagent for the analysis.
- They are not present in natural waters.
- They are not significantly toxic and do not affect animal life.

5.2.2.4 DISADVANTAGES OF RHODAMINES

- Not very soluble.
- Rather expensive.

- The characteristics of certain rhodamines are affected by sunlight, temperature or the presence of certain silts in water (problems due to adsorption effects).
- Residual colour is difficult to remove.

5.2.2.5 ADVANTAGES OF SODIUM CHLORIDE

- The resistivity of the sodium chloride solution is proportional to concentration over a wide range.
- It is relatively cheap.

5.2.2.6 DISADVANTAGE OF SODIUM CHLORIDE

- It cannot be used at very low concentrations.

6 CHOICE OF MEASURING LENGTH AND ADEQUATE MIXING DISTANCE

6.1 Introduction

When a tracer is used to measure the flow of water in a conduit, there should be sufficient distance between the region in which the tracer is injected or produced and the region in which concentration or transit time measurements are made. The distance which is required in order to allow the tracer to mix with the water in the conduit is known as the mixing distance.

The mixing distance is defined as the shortest distance at which the maximum variation (x), over the cross-section, of $\int_0^t C_2 dt$ for the integration method is less than some predetermined value (for example 0,5 %); this corresponds to the concentration of tracer for the constant rate injection method. Thus, the mixing distance is not a fixed value, but varies according to the allowed concentration variations : the smaller the acceptable variation the greater the mixing distance.

For highest accuracy in flow measurement it is necessary to ensure the smallest possible values of x at the measuring cross-section. However, in practice higher values of x may have to be tolerated when sufficiently long lengths of conduit are not available.

A multipoint sampling or detection arrangement should be used where possible, particularly when a systematic variation in concentration or in $\int_0^t C_2 dt$ may exist at the sampling cross-section.

Depending on the tracer and the method of detection used, the mixing requirements for the transit time method may not be so stringent as for dilution methods.

Several techniques have been developed to reduce the mixing distance and these should be used whenever possible (see 6.3).

6.2 Mixing distance

6.2.1 *Theoretical derivation of mixing distance*

6.2.1.1 CENTRAL INJECTION

The following equations relating mixing distance (L/D) in terms of the varying concentration of tracer across the conduit, Reynolds number (Re) and pipe friction, have been developed. Equation (1) is derived on the basis of a constant radial diffusion coefficient and uniform flow velocity; equation (2) is derived on the basis of a parabolic distribution of radial diffusion coefficient and uniform flow velocity; equation (3) assumes a parabolic distribution of radial diffusion coefficient and a logarithmic velocity profile.

$$\frac{L}{D} = 1{,}18\sqrt{\frac{8}{\lambda}}\left(2{,}94 - \frac{\ln x}{2{,}30}\right) \qquad \dots (1)$$

$$\frac{L}{D} = \left(2{,}95 - \frac{\ln x}{2{,}4}\right)\sqrt{\frac{8}{\lambda}} \qquad \dots (2)$$

$$\frac{L}{D} = \left(20{,}5 - 2{,}85 \ln x\right) Re^{1/10}\left[\frac{\lambda_{\text{smooth}}}{\lambda_{\text{pipe}}}\right]^{1/2} \qquad \dots (3)$$

where

x is the maximum variation, in percent, across the pipe, of concentration C_2 for the constant rate injection method, or of $\int_0^t C_2 dt$ for the integration method, at a distance L from the point of injection;

D is the diameter of the conduit;

λ is the specific coefficient of resistance of the conduit.

The above equations presented graphically in figure 1 show the increase in mixing distance with decrease of x for a Reynolds number of $Re = 10^5$ and a smooth conduit.

The slight dependance of mixing distance on Reynolds number (see equation (3), for example) is shown in figure 2. For a change in Re from 10^5 to 10^6, at $x = 1$ %, the mixing distance increases by only 25 %, approximately.

6.2.1.2 RING INJECTION

For uniform injection over a ring with a radius of 0,63 of the conduit radius, the mixing distances are reduced to about one third of the values derived for a central injection.

6.2.2 *Experimental derivation of mixing distance*

Values of mixing distance obtained experimentally for a central injector in an unobstructed, straight, circular conduit are about twice the values predicted theoretically. This difference is attributed to several causes but particularly to the difference between the actual flow conditions and those assumed in the theoretical analysis. Care shall therefore be exercised in the treatment of theoretical results.

The measured change in mixing distance with x for a central injection and for three other methods of injection is shown as an example in figure 3. It should be noted that the flow turbulence level influences these results.

6.3 Examples of methods of reducing mixing distance

6.3.1 *Multi-orifice injectors*

When the tracer is injected equally through a number of orifices spaced across the conduit, a reduction in mixing distance can be achieved compared with the mixing distance associated with a central injector.

An example of the reduction in mixing distance that can be achieved by using four injectors, equally spaced around the wall of a conduit and a radius of 0,63 for the conduit radius, is shown in figure 3.

6.3.2 *High velocity jets*

If the tracer is injected against the flow with a velocity which far exceeds the mean velocity of the water in the conduit, impact mixing occurs at the termination of the jet. The reduction in mixing distance depends on the number and the momentum of the jets and their inclination to the direction of flow.

Accurate quantitative information on the effect of the above parameters is not yet available but reductions in mixing distance to approximately 30 % of that for a single central injector can be obtained by using a simple configuration of jets.

6.3.3 *Vortex generators*

A turbulent wake which stimulates mixing and reduces the mixing distance can be obtained by flow deflection plates built into the conduit close to the injection region.

For example, the mixing distance has been reduced to one-third of that with a single central injector by injecting the tracer through three triangular plates set at an angle of 40° to the direction of flow.[1]

6.3.4 *Pumps and turbines*

A considerable reduction in mixing distance may be effected by injecting the tracer upstream of a pump or turbine. Information on mixed-flow pumps indicates that this type of pump reduces the mixing distance by about 100 diameters.

6.3.5 *Bends, valves and other obstructions*

Obstructions in the conduit introduce additional turbulence and thus tend to reduce the mixing distance. Quantitative information on these types of mixing promotors is not available but measuring sections that include these devices are to be preferred. In the transit time method, however, the length of conduit between detectors should be straight and free of obstruction if the highest accuracy is required.

6.4 Multi-orifice sampling

The error due to the use of a distance between the injection cross-section and the sampling cross-section less than the mixing distance can be reduced if samples are recovered simultaneously from a number of positions across the conduit and are then mixed prior to measurement.

For example, at $Re = 10^5$, six sampling points having the same discharge equally spaced across the conduit at 50 diameters downstream from the point of injection were found to be equivalent to a single sampling point at the wall at about 100 diameters from the point of injection.

6.5 Choice of measuring length

6.5.1 *Dilution methods*

For constant rate injection and integration methods, the measuring length is defined as the length of conduit between the injection position and the sampling point. This length should be determined after consideration has been given to 6.2, 6.3 and 6.4 about the mixing of the tracer.

The addition of fluid of the same nature as that in the measuring length does not affect the result, provided that the mixture remains homogeneous at all points of the measuring cross-section. The flow measured is the total flow through the measuring cross-section.

If the measuring length includes losses or sampling points, the result obtained is valid only if it is possible to estimate or to verify that the mixture is homogeneous upstream of the loss zone. In this case, the flow measured is the flow in the conduit immediately upstream of the loss zone.

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[1] See on this subject *Triangular plate vortex generators in flow measurement* by G.V. Evans (Fluid Flow – March 1968 – Vol. 13 – No. 3 – pp. 375 – 376).

6.5.2 *Transit time methods*

The measuring length for the transit time methods may be considered to consist of two parts, i.e. :

a) the length of conduit between the point of injection of the tracer and the first detector position;

b) the length of conduit between the two detector positions.

Part a) should be determined after consideration has been given to 6.2, 6.3 and 6.4 about the mixing of tracer. The mixing requirement for these methods may not be so stringent as for the dilution methods, depending on the tracer and method of detection used.

Part b) should be determined from consideration of the degree of longitudinal dispersion of the tracer, the mean velocity expected and the accuracy of measurement of transit time.

Additions of fluid of the same nature as the fluid in the measuring length do not affect the result, provided that the mixture remains homogeneous at all points of the cross-section in which the first detector is positioned.

Losses of fluid from the conduit before the first detection position do not affect the result but if the tracer is not completely mixed at the position of the loss, the amplitude of the concentration-time distribution at the detection positions may be affected and its value changed by a constant factor.

Losses or additions of fluid in the length of conduit between detection positions would cause serious errors in the measurement of flow. Consequently it is essential that the conduit between the two detector positions contains no branch connections and that there is no leakage from it.

For the highest accuracy, the length of conduit between detector positions should be straight and free from obstruction (for example valves).

In all cases it is necessary to know the volume of conduit between the detector positions.

7 ERRORS

7.1 General

As with any measurement of a physical quantity, the determination of a flow rate in a conduit by tracer methods is subject to uncertainties related either to systematic errors due to errors in the measuring apparatus or in the measuring process used, or to a random error obtained by random variations in the flow system, or in the measuring equipment.

7.2 Systematic errors

7.2.1 As with any measuring instruments, those used for flow measurement may possess a number of imperfections, linked to their design or to the selected method of use, which affect the measurements in a known manner. These effects may be eliminated totally or partly, either by using appropriate methods or by applying corrections to the results based on a knowledge of some environmental parameters of the measuring process. For example, the "zero method" can be used to eliminate the effects due to variation in the indications of colorimeter photoelectric cells with changing ambient temperature; similarly, corrections can be applied to the results of measurement of flow rate by means of an orifice plate made at a temperature different from that at calibration or reference conditions.

Errors of the type described above may be reduced by the use of appropriate instruments; in such cases, estimation is based on a knowledge of the physical laws affecting the instrument.

7.2.2 Another type of systematic error can exist in the measurement of flow by means of tracers, of which the direction may be defined but the magnitude cannot be estimated; such errors result from phenomena connected with the dissolution of tracer in water and, particularly, with the mixture and possible disappearance or transformation of the injected product.

The use of the mixing condition given in 6.1 leads to the acceptance of an error in the result which is generally not systematic except in a few cases, as, for example, when the density of the tracer used is very different from that of the water the flow of which is to be measured. It is then necessary to take great care that good mixing is achieved by multiple sampling or detection points in the sampling cross-section.

In some cases, the injected tracer is liable to react with the water circulating in the conduit, or with any substance with which it may come into contact between the injection and detection times. In general, when using dilution methods, the systematic errors which may be caused by these reactions lead to an overestimation of the flow rate (disappearance of tracer). This error can be reduced to insignificance by the selection of a suitable tracer, and the use of an appropriate injection, detection, sampling and analysis procedure.

7.3 Random errors

In this International Standard, error values correspond to 95 % confidence limits.

7.3.1 The possible error on a flow rate measurement cannot be determined exactly *a priori*, but it is possible :

a) to achieve statistical estimation of the tolerance (for 95 % confidence limits), under general conditions of use of a particular method, by an evaluation of the tolerances on individual measurements in the calculation of rate of flow (this estimation is made possible by an analysis of a large number of measurements);

b) in the particular case of a measurement repeated only a small number of times, to determine 95 % confidence limits applicable to the estimate of the result that would have been obtained if the measurements had been repeated for a large number of times as obtained from the mean of the measuring sample alone (use of the Student variable).

7.3.2 *Tolerance under general conditions of use*

The tolerance may be assumed to be twice the standard deviation of the measurement of flow rate in the case of Gaussian distribution of errors. If the individual errors in the various measurements are small and independent, the tolerance on flow measurement is then equal to :

$$2\frac{\sigma_{q_v}}{q_v} = 2\sqrt{\left(\frac{\partial q_v}{\partial x_1} \times \frac{\sigma_{x_1}}{q_v}\right)^2 + \left(\frac{\partial q_v}{\partial x_2} \times \frac{\sigma_{x_2}}{q_v}\right)^2 + \dots}$$

where

$\frac{\partial q_v}{\partial x_1}, \frac{\partial q_v}{\partial x_2}, \dots$ are partial derivatives, the value of which depend on the manner in which q_v is a function of $x_1, x_2, \dots$;

$x_1, x_2, x_3, \dots$ are the independent quantities measured;

$\sigma_{x_1}, \sigma_{x_2}, \sigma_{x_3}, \dots$ are the standard deviations of the measurements of $x_1, x_2, x_3, \dots$

If an independent quantity y has been obtained by N repeated measurements, which would be the case where, for example, a dilution ratio has been obtained from a large number of samples, and if the results of these measurements are denoted by $y_1, y_2, \dots, y_n$, the standard deviation of the mean y_0 of these N measurements is defined as :

$$\sigma_{y_0} = \sqrt{\frac{\sum_{i=1}^{i=N} (y_1 - y_0)^2}{N(N-1)}}$$

If the value of an independent quantity results from a small number of measurements N it is possible within a given confidence limit to define an interval in which the standard deviation of a large number of measurements can be found :

$$\frac{s\sqrt{N}}{\chi_{(n+100)/2}} < \text{standard deviation} < \frac{s\sqrt{N}}{\chi_{(100-n)/2}}$$

where

s is the estimate of the standard deviation calculated from a small number of measurements;

n is the confidence limit (in %) chosen;

$\chi_{(n+100)/2}$ and $\chi_{(100-n)/2}$ are the numerical values derived from a distribution table of "χ^2" (see table in annex).

The value of n is generally taken to be 95.

Example : the 95 % confidence interval of the standard deviation of a quantity, the estimation of which from 19 measurements is s, is :

$$\frac{s\sqrt{19}}{\sqrt{8{,}23}} \quad \frac{s\sqrt{19}}{\sqrt{31{,}5}} \quad \text{i.e. } 1{,}5s \text{ and } 0{,}78s$$

where 8,23 and 31,5 are respectively the values read in the distribution table of χ^2 :

$$\left.\begin{aligned} &\text{column } P = 0{,}975 = \frac{95+100}{2\times 100} \\ &\text{column } P = 0{,}025 = \frac{100-95}{2\times 100} \end{aligned}\right\} \text{line } \nu = 18 = 19 - 1$$

7.3.3 *Estimation of the tolerance on a particular measurement repeated a small number of times*

The 95 % confidence interval of the mean value is :

$$\overline{q_v} \pm t^* \frac{s}{\sqrt{N-1}}$$

where

$\overline{q_v}$ and s are the estimates of the mean and of the standard deviation respectively as obtained from the measurements;

t^* is the numerical value of the Student variable derived from a distribution table;

ν is the degree of freedom.

An extract from this table is given below.

ν	t^*
1	12,706
2	4,303
3	3,182
4	2,776
5	2,571
6	2,447
8	2,306
10	2,228
15	2,131
20	2,086
25	2,060
30	2,042
40	2,021
80	2,000
120	1,980
∞	1,960

7.3.4 The error of the measurement is obtained from the equation given in 7.3.3 and should be compared with the error estimated from the equation in 7.3.2 in order to check that the method has been correctly applied. If a considerable difference in values is obtained, the results should be re-examined in order to determine the source of the discrepancy.

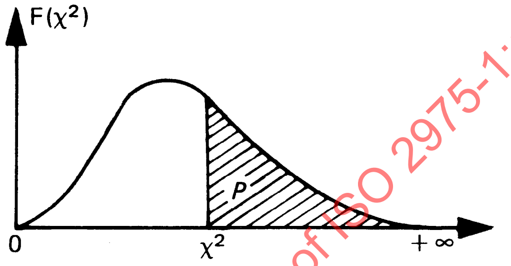
ANNEX

DISTRIBUTION TABLE OF χ^2

(Pearsons' Law)

Values of χ^2 having the probability P to be exceeded with ν degrees of freedom

$$P = \frac{n + 100}{2 \times 100} \text{ or } \frac{100 - n}{2 \times 100}$$



When $\nu \leqslant 30$

ν \ P	0,990	0,975	0,950	0,900	0,100	0,050	0,025	0,010	0,001
1	0,0002	0,0010	0,0039	0,0158	2,71	3,84	5,02	6,63	10,83
2	0,02	0,05	0,10	0,21	4,61	5,99	7,38	9,21	13,82
3	0,12	0,22	0,35	0,58	6,25	7,81	9,35	11,34	16,27
4	0,30	0,48	0,71	1,06	7,78	9,49	11,14	13,28	18,47
5	0,55	0,83	1,15	1,61	9,24	11,07	12,83	15,09	20,52
6	0,87	1,24	1,64	2,20	10,64	12,59	14,45	16,81	32,46
7	1,24	1,69	2,17	2,83	12,02	14,07	16,01	18,47	24,32
8	1,65	2,18	2,73	3,49	13,36	15,51	17,53	20,09	26,13
9	2,09	2,70	3,33	4,17	14,68	16,92	19,02	21,67	27,88
10	2,56	3,25	3,94	4,87	15,99	18,31	20,48	23,21	29,59
11	3,05	3,82	4,57	5,58	17,27	19,67	21,92	24,72	31,26
12	3,57	4,40	5,23	6,30	18,55	21,03	23,34	26,22	32,91
13	4,11	5,01	5,89	7,04	19,81	22,36	24,74	27,69	34,53
14	4,66	5,63	6,57	7,79	21,06	23,68	26,12	29,14	36,12
15	5,23	6.20	7,26	8,55	22,31	25,00	27,49	30,58	37,70
16	5,81	6,91	7,96	9,31	23,54	26,30	28,84	32,00	39,25
17	6,41	7,56	8,67	10,08	24,77	27,59	30,19	33,41	40,79
18	7,01	8,23	9,39	10,86	25,99	28,87	31,53	34,80	42,31
19	7,63	8,91	10,12	11,65	27,20	30,14	32,85	36,19	43,82
20	8,26	9,59	10,85	12,44	28,41	31,41	34,17	37,57	45,32
21	8,90	10,28	11,59	13,24	29,61	32,67	35,48	38,93	46,80
22	9,54	10,98	12,34	14,04	30,81	33,92	36,78	40,29	48,27
23	10,20	11,69	13,09	14,85	32,01	35,17	38,08	41,64	49,73
24	10,86	12,40	13,85	15,66	33,20	36,41	39,37	42,98	51,18
25	11,52	13,12	14,61	16,47	34,38	37,65	40,65	44,31	52,62
26	12,20	13,84	15,38	17,29	35,56	38,88	41,92	45,64	54,05
27	12,88	14,57	16,15	18,11	36,74	40,11	43,19	46,96	55,48
28	13,57	15,31	16,93	18,94	37,92	41,34	44,46	48,28	56,89
29	14,26	16,05	17,71	19,77	39,09	42,56	45,72	49,59	58,30
30	14,95	16,79	18,49	20,60	40,26	43,77	46,98	50,89	59,70